

Module-2

Image Transforms

Introduction to Image Transforms

• **Definition:** A mathematical tool that represents an image in a different domain (e.g., frequency domain).

• **Need for Transforms:**

- **Mathematical Convenience:** Simplifies complex operations like [convolution into simple multiplication](#).
- **Data Compression:** Packs most image energy into a few coefficients.
- **Feature Extraction:** Highlights edges or patterns for recognition.
- **Fast Computation:** Enables efficient algorithms like the Fast Fourier Transform

Two-Dimensional Orthogonal & Unitary Transforms

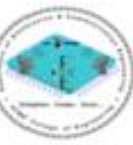
General Formulation: A 2D transform maps an image $f(x,y)$ to a set of coefficients $V(u,v)$ using a forward transformation kernel.

- **Unitary Transform:** A linear transform where the transformation matrix A satisfies $A^{-1} = A^H$ (conjugate transpose).
- **Orthogonal Transform:** A special case where the matrix A is real and unitary ($A^{-1} = A^T$).
- **Separability:** If the 2D kernel can be split into two 1D kernels, the transform can be computed row-by-row then column-by-column, greatly reducing computation time.

Properties of Unitary Transforms

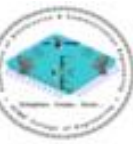
- **Energy Conservation:** Total signal energy is preserved ($\|f\|^2 = \|V\|^2$), according to the [2D Parseval relation](#).
- **Energy Compaction:** Most of the image information is concentrated into a few small-index coefficients near the origin.
- **De-correlation:** Highly correlated input pixels are transformed into [uncorrelated output coefficients](#), facilitating easier compression.
- **Rotation Invariance:** A unitary transform acts as a rotation of the image vector in N^2 -dimensional space.

2-D Discrete Fourier Transform (DFT)



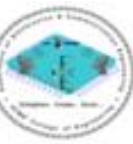
- **Concept:** Represents an image as a sum of **sinusoidal basis images**.
- **Key Features:**
 - **Complex Values:** Provides both magnitude and phase information.
 - **Periodicity:** The transform and its inverse are periodic.
 - **Applications:** Frequency domain filtering, noise removal, and texture analysis.

Discrete Cosine Transform (DCT)



- **Definition:** Uses only cosine basis functions and produces real coefficients.
- **Advantages:**
 - **Excellent Energy Compaction:** Close to the optimal Karhunen-Loève Transform (KLT) for most images.
 - **Real and Fast:** Avoids complex numbers and has efficient "fast" implementations.
- **Primary Use:** The standard for [lossy image compression](#) (e.g., JPEG)

Harr Transform



- **Mechanism:** A [simple wavelet transform](#) using square-shaped basis functions.
- **Mathematical Operations:** Uses only simple [additions and subtractions](#) to decompose data into averages and differences.
- **Strengths:**
 - Extremely fast to compute.
 - Excellent for detecting sharp transitions or sudden changes in an image.
 - Provides multi-resolution representation.

Conclusion & Comparison



- **DFT** is best for frequency analysis and filtering.
- **DCT** is superior for compression due to energy compaction.
- **Haar** is ideal for fast, multi-scale edge detection and analysis